

VK Multimedia Information Systems

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Dienstags, 16.00 Uhr c.t., E.1.42

Indexing



- Spatial Indexes
- MDS - FastMap
- Hashing
- Metric Indexes
- Inverted Lists



Indexing Visual Information



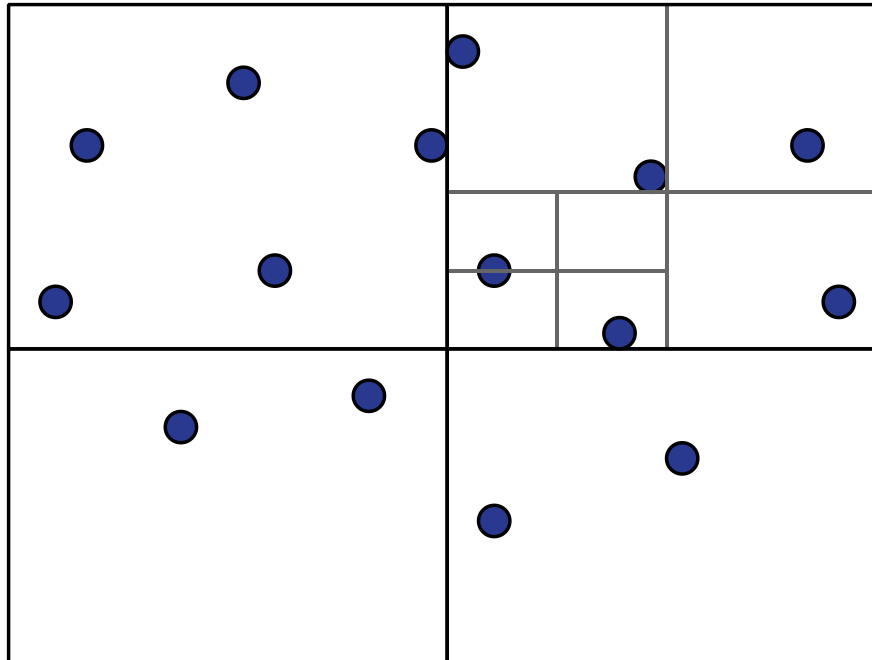
- Text is indexed in inverted lists
 - Search time depends on # of terms
- Visual information expressed by “vectors”
 - Combined with a metric capturing the semantics of similarity
 - Inverted list does not work here
 - An “index of vectors” is needed

Indexing Visual Information



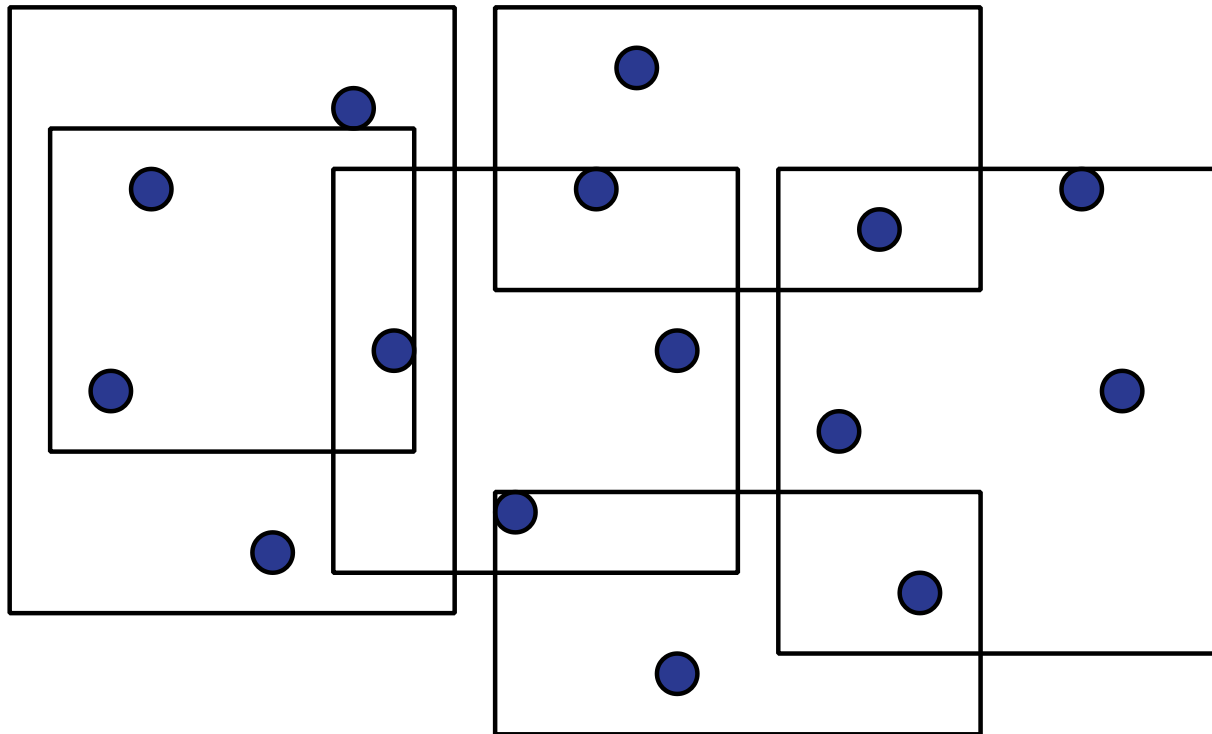
- Vectors describe “points in a space”
 - Space is n -dimensional
 - n might be rather big
- Distance (metric) between points
 - E.g. L1 or L2 ...
- Query is also a vector $:=$ point
 - Searching for points (vectors) near to query
- Idea for index:
 - Index neighborhood ...

Spatial Indexes



Using equally sized rectangles (Optimal for L1 ...)

Spatial Indexes



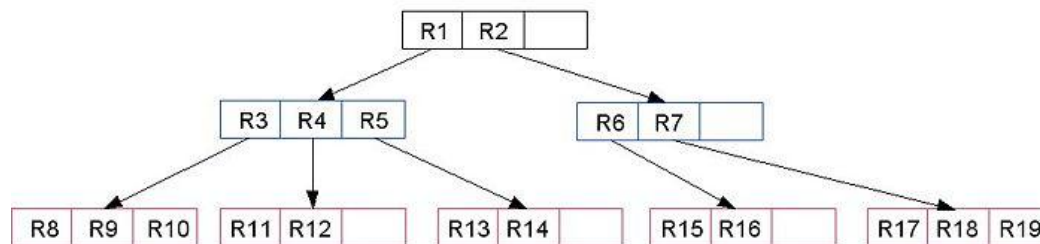
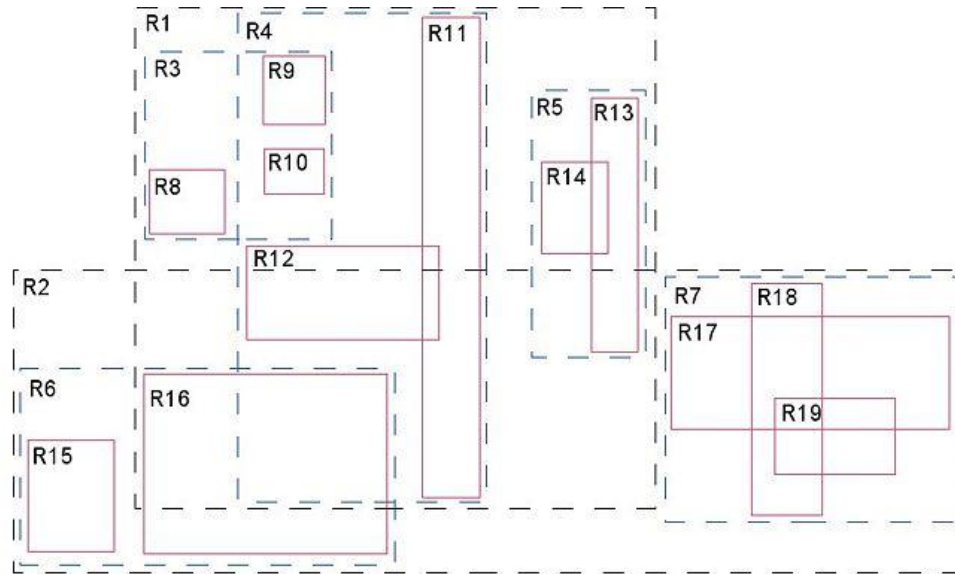
Using overlapping rectangles ...

Spatial Indexes

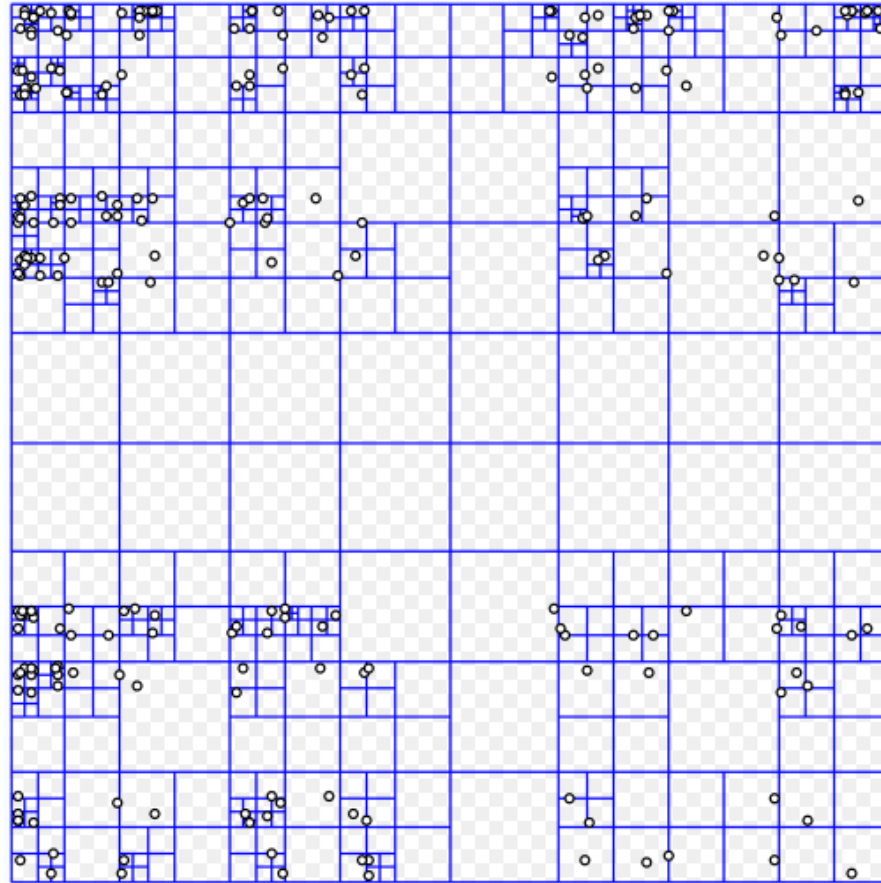


- Common data structures
 - R Tree
 - R^* , R^+ ,
 - Overlapping rectangles
 - Search is a rectangle
 - Quadtree (Octtree)
 - Equally sized regions, subdivided
 - 4 quadrants or 8 octants
 - Search selects quadrants

R-Tree



Quadtree



Spatial Indexes: Drawbacks



- Data structures must minimize
 - false negatives (-> maximizes recall)
 - false positives (-> search time)
- Features, distance function & parameters need to be selected at index time
 - Search combining multiple descriptors is complicated issue
- Works best for spaces with small dimension n
 - MDS has to be applied ...

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Multidimensional Scaling (MDS)



- Reducing the dimensions of a feature space
 - E.g. From 64 dimensions to 8
 - Without losing too much information about neighborhoods
- Applications in multimedia retrieval
 - Indexing based on coordinates
 - Spatial Indexes:
 - Data structures to find nearest neighbors fast

Multidimensional Scaling (MDS)



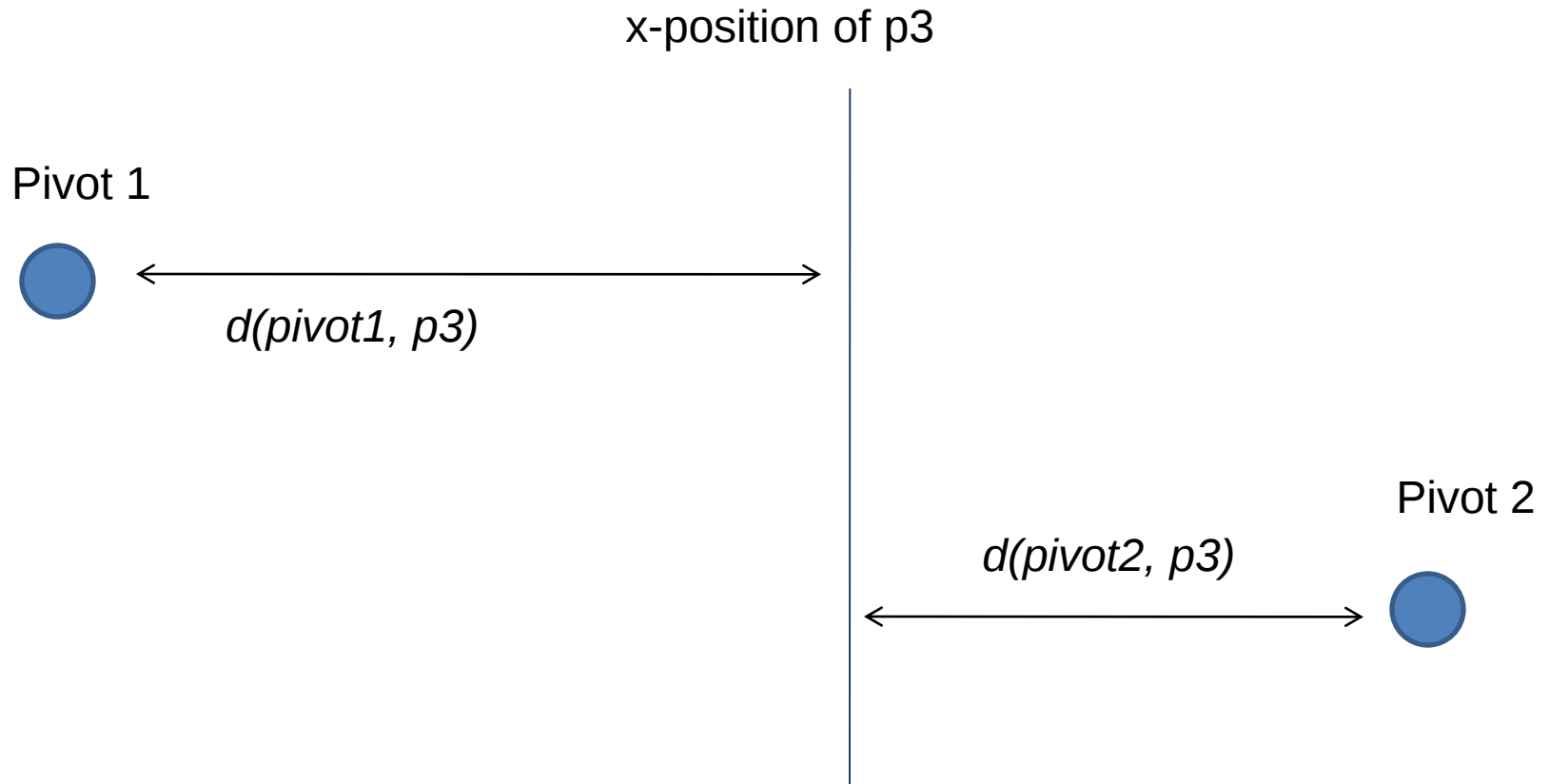
- Interpolation: FastMap
 - Linear in terms of objects
 - Used e.g. in IBM QBIC
- Iterative: Force Directed Placement
 - Iterative optimization of initial placement
 - Cubic runtime

FastMap

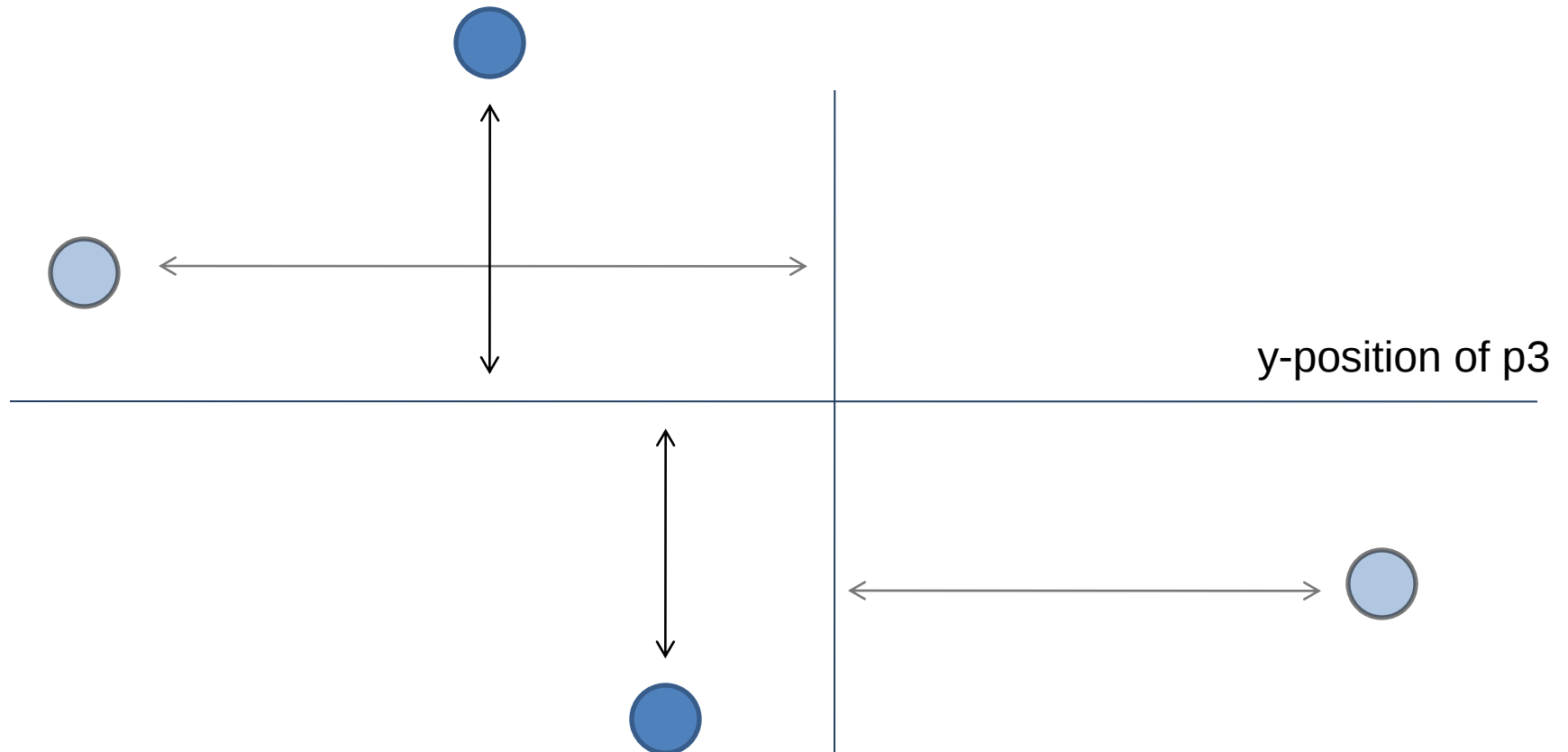


- For Each dimension d
 - Find Pivots (the most distant objects)
 - For each object, which is not a pivot
 - Interpolate position between pivots in this dimension
 - Next object
- Next Pivot

FastMap



FastMap



FastMap: Pivots



How to find optimal pivots?



- Select one object randomly $\rightarrow P_1$
- Select Object P_2 with maximum distance from P_1 to P_2
- If $d(P_1, P_2) < t$
 - Set $P_1 = P_2$
 - Goto (2)

Normally no threshold is used but this is done x times.

Force Directed Placement



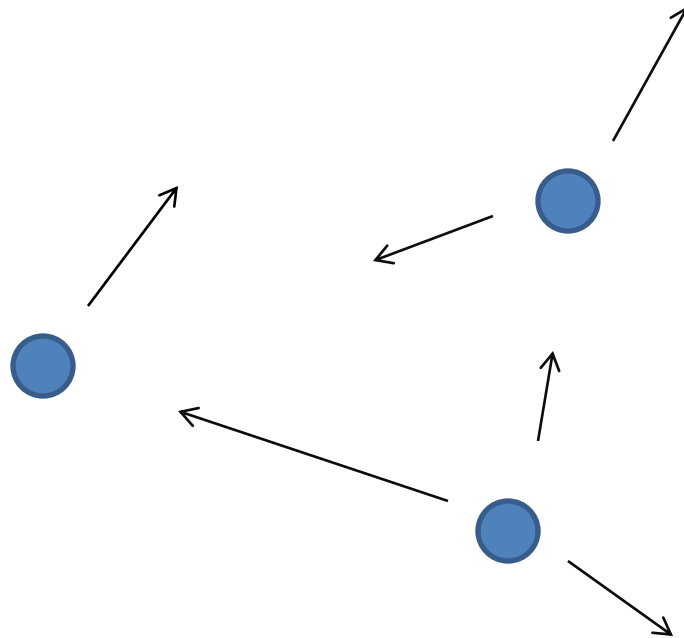
1. All objects are assigned coordinates
2. For each object o
 - Movement vector $v = 0$
 - For each object p
 - Calculate repulsion & attraction forces between o & p
 - Compute movement vector $v(o, p)$ depending on the forces
 - $v = v + v(o, p)$
3. If overall movement is still high goto 2.

FDP: Parameters



- Gravity as overall attraction
 - Prevents uncontrolled spread
- Overall repulsion
 - Prevents coming objects from coming too close
- Minimum distance
 - If objects are on the coordinates
- Spring parameters
 - Repulsion stronger close up
 - Attraction stronger if far away

FDP



Demo



- Emir

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Locality Sensitive Hashing (LSH)



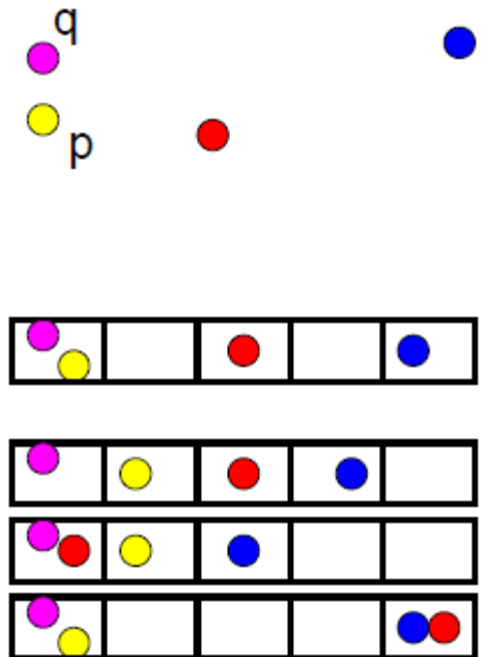
- Algorithm to determine the “Approximate Near(est) Neighbor”
- Given: a set P of points in R^d
- Nearest Neighbor: query q returns point $p \in P$ minimizing $|p-q|$
- r-Near Neighbor: query q returns point $p \in P$ so that $|p-q| \leq r$

src. <http://people.csail.mit.edu/indyk/mmds.pdf>

LSH - Idea



- Construct hash functions $g: \mathbb{R}^d \rightarrow U$ so that
 - If $|p-q| \leq r$ then $\Pr[g(p)=g(q)]$ is “not so small”
 - If $|p-q| > cr$ then $\Pr[g(p)=g(q)]$ is “small”



LSH - Process



- A family H of hash functions $h: \mathbb{R}^d \rightarrow U$ is called $(P1, P2, r, cr)$ -sensitive if
 - If $|p-q| \leq r$ then $\Pr[h(p)=h(q)] > P1$
 - If $|p-q| > cr$ then $\Pr[h(p)=h(q)] < P2$
- LSH uses functions $g(p) = \langle h_1(p), \dots, h_k(p) \rangle$
 - Preprocessing
 - Select functions $g_1 \dots g_L$
 - Hash all $p \in P$ to buckets $g_1(p) \dots g_L(p)$
 - Query
 - Retrieve points from buckets $g_1(q) \dots g_L(q)$



- LSH solves c -approximate NN with:
 - Number of hash functions: $L = n^\rho$,
 $\rho = \log(1/P_1) / \log(1/P_2)$
 - E.g., for the Hamming distance we have $\rho = 1/c$
 - Constant success probability per query q

LSH - Random Projection



- Approximates cosine distance
 - One bit per hash function
- For two feature vectors
 - number of matching bits is proportional to the probability of matching

LSH - Random Projection



- Hash function
 - $h(v) = \text{signum}(v \cdot r)$
 - whereas r is a random hyperplane
- $h(v)$ is in $\{-1, 1\}$
 - so $h(v)$ produces 1 bit.
- k hash functions produce a k bit hash value

LSH - Random Projection



- Once:
 - generate k hyperplanes (uniform distribution)
 - with d dimensions each
- At index time
 - compute hash for each feature
- At search time
 - filter index based on hashes
 - at least m bits must match

LSH - Stable Distribution



- Approximate other metrics

$$h_{\mathbf{a},b}(\mathbf{v}) = \left\lfloor \frac{\mathbf{a} \cdot \mathbf{v} + b}{r} \right\rfloor$$

- Entries of $\mathbf{a}$ are drawn from stable distribution.
- b is real number drawn from $[0, r)$ with uniform distribution.
- Gaussian for $\mathbf{a} \rightarrow \text{L2}$

LSH - Stable Distribution



- Employ Box-Muller transformation
 - U_1, U_2 drawn from $[0,1]$ with uniform distribution

$$Z_0 = R \cos(\Theta) = \sqrt{-2 \ln U_1} \cos(2\pi U_2)$$

$$Z_1 = R \sin(\Theta) = \sqrt{-2 \ln U_1} \sin(2\pi U_2).$$

- Z_0, Z_1 are independent random variables with standard normal distribution.

LSH - Stable Distribution



```
private static double drawNumber() {
    return Math.sqrt(-2 * Math.log(Math.random()))
        * Math.cos(2 * Math.PI * Math.random());
}

public static void generateHashFunctions() throws IOException {
    ObjectOutputStream oos = new ObjectOutputStream(new FileOutputStream(name));
    oos.writeInt(dimensions);
    oos.writeInt(numFunctionBundles);
    for (int c = 0; c < numFunctionBundles; c++) {
        oos.writeDouble(Math.random() * binLength);
    }
    for (int c = 0; c < numFunctionBundles; c++) {
        for (int j = 0; j < dimensions; j++)
            oos.writeDouble(drawNumber());
    }
    oos.close();
}
```

LSH - Stable Distribution



- Generate hashes from histogram

```
public static int[] generateHashes(double[] histogram) {
    double product = 0d;
    int[] result = new int[numFunctionBundles];
    for (int k = 0; k < numFunctionBundles; k++) {
        for (int i = 0; i < histogram.length; i++) {
            product += histogram[i] * hashA[k][i];
        }
        result[k] = (int) Math.floor((product + hashB[k])/binLength);
    }
    return result;
}
```

LSH - Stable Distribution



- For histograms of dimension d
 - if m of k hash functions match
 - $m \cdot k$ has to be greater than d
- Bin length r defines the length of hashes
 - Elements of $\mathbf{a}$ & $\mathbf{v}$ are bounded
 - Therefore hash is bounded

$$h_{\mathbf{a},b}(\mathbf{v}) = \left\lfloor \frac{\mathbf{a} \cdot \mathbf{v} + b}{r} \right\rfloor$$

LSH - Stable Distribution



- CEDD with $r=1$
 - On Wang Simplicity: min = -203, max = 167
- CEDD with $r=0.1$
 - On Wang Simplicity: min = -1746, max = 2130
- Smaller r leads to a lot more bins
 - Necessary for trade-off time vs. space

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Metric Indexes



A metric index is a *tree of nodes*

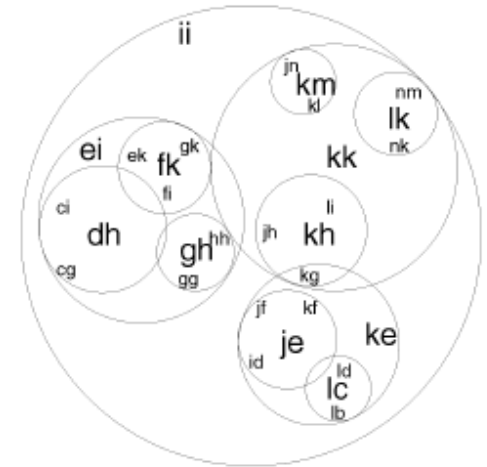
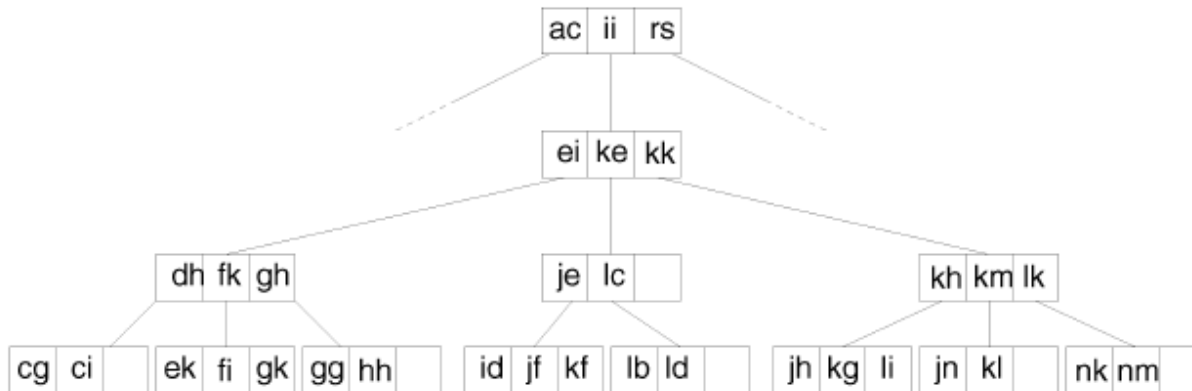
- Each node containing a fixed maximum number of entries
- Each entry is constituted by a routing entry D

src. Berretti, S.; del Bimbo, A. & Vicario, E.
Efficient Matching and Indexing of Graph Models in Content-Based Retrieval
IEEE Transactions on Pattern Analysis and Machine Intelligence, **2001**, 23, 1089-1105

Metric Indexes



- D is the root of a sub index in the *covering region* of D
 - Also defines a radius r_D being the maximum distance from D to any entry in the covering

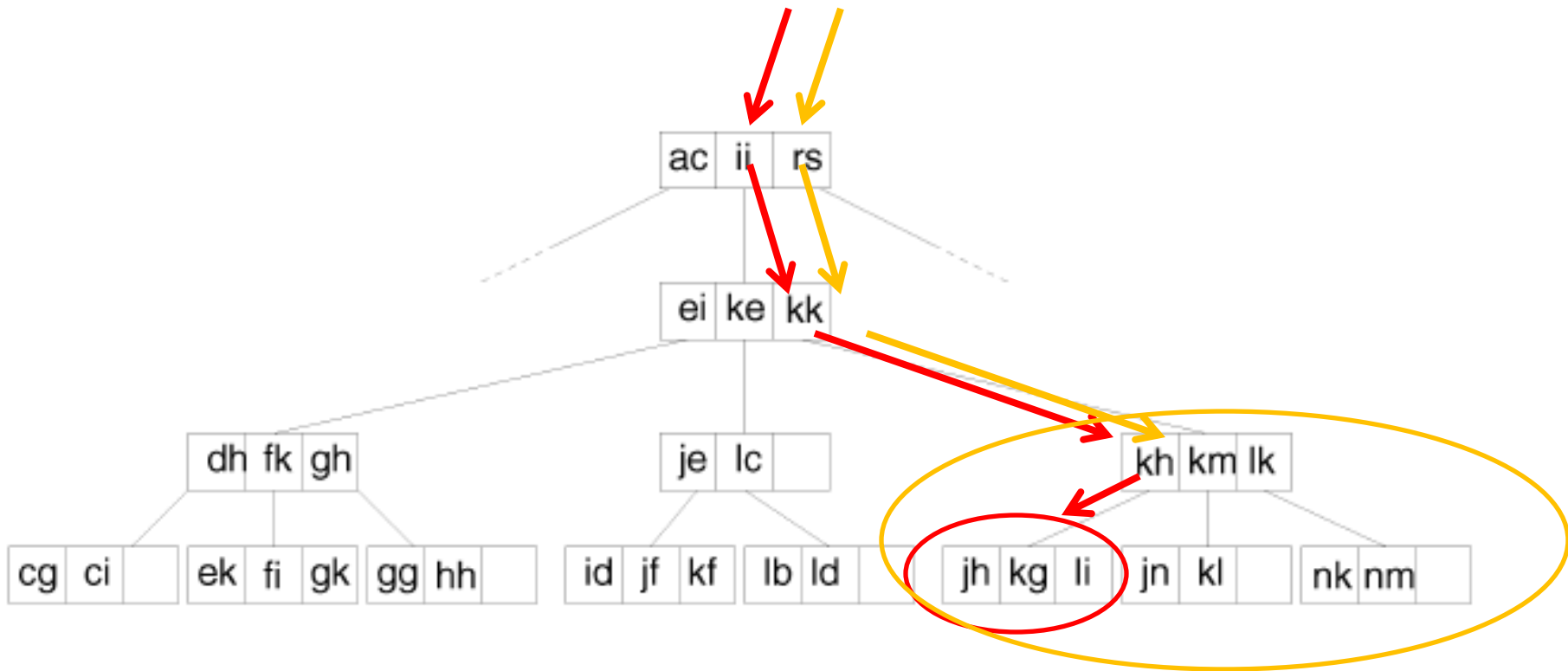


Metric Index: Construction



- Top-down: Indexing an entire archive at once
 - All documents to index are known
 - No iterative additions
- Bottom-up: Indexing on insertion
 - Documents are indexed as they are added to the collection
 - Optimizations (e.g. splitting) have to be done

Metric Index: Searching



Indexing



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Metric Spaces



src. G. Amato & P. Savino, „Approximate
Similarity Search in Metric Spaces Using Inverted Files“,
Infoscale 2008

- $\mathcal{M} = (\mathcal{D}, d)$
 - Data domain $\mathcal{D}$
 - *Total (distance) function* $d: \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}$ (metric function or metric)
- The metric space postulates:
 - Non negativity $\forall x, y \in \mathcal{D}, d(x, y) \geq 0$
 - Symmetry $\forall x, y \in \mathcal{D}, d(x, y) = d(y, x)$
 - Identity $\forall x, y \in \mathcal{D}, x = y \Leftrightarrow d(x, y) = 0$
 - Triangle inequality $\forall x, y, z \in \mathcal{D}, d(x, z) \leq d(x, y) + d(y, z)$

Similarity Search in Metric Spaces

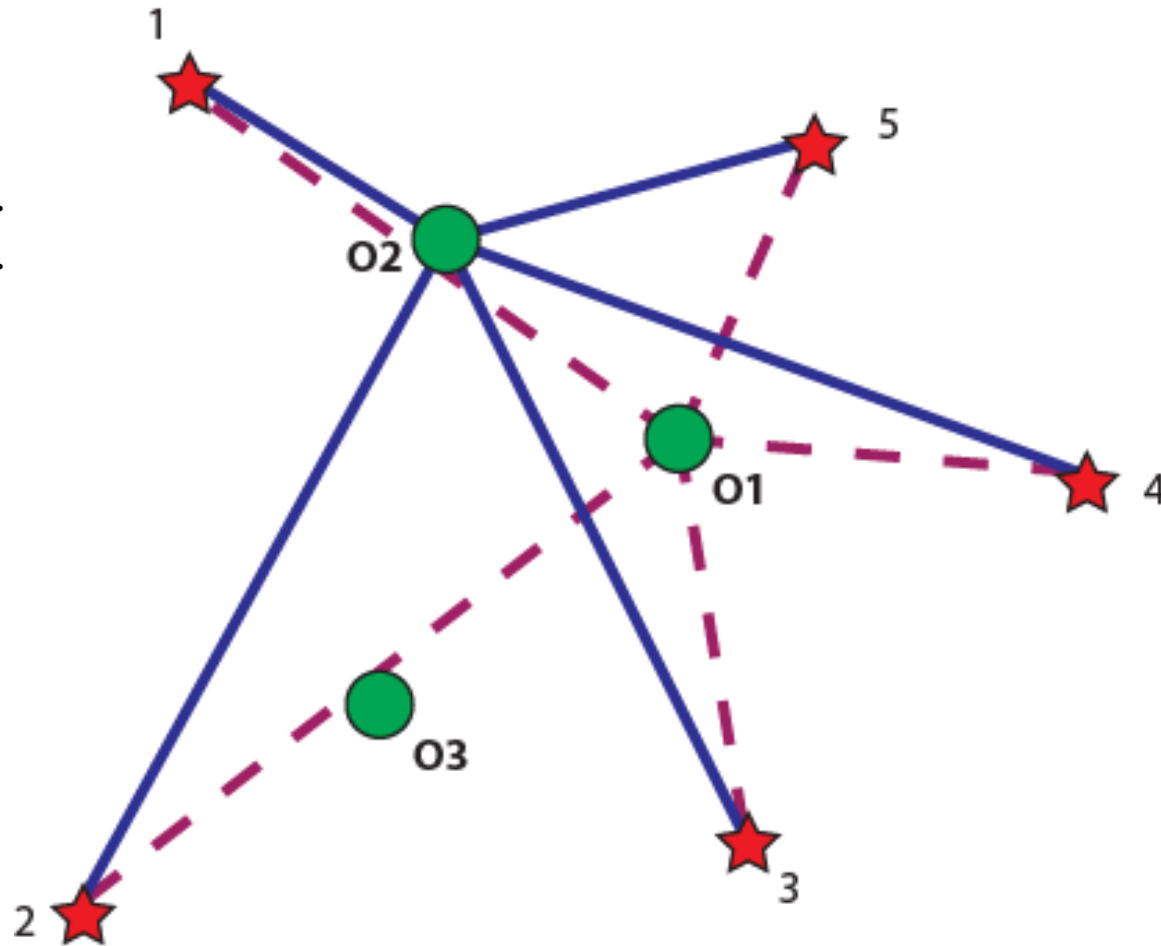


- Objects close to one another see the space in a “similar” way
- Choose a set of *reference objects* RO
- Orderings of RO according to the distances from two similar *data objects* are similar as well
 - Represent every data object o as an ordering of RO from o
 - Measure similarity between two data objects by measuring the similarity between the corresponding orderings

Similarity Search in Metric Spaces



O1 := <5, 3, 4, 1, 2>
O2 := <1, 5, 3, 4, 2>
O3 := ...



Similarity Search in Metric Spaces



- Spearman Footrule Distance

$$SFD(S_1, S_2) = \sum_{ro \in RO} |S_2(ro) - S_1(ro)|$$

Similarity Search in Metric Spaces



[Slides G. Amato ...]

Thanks ...

